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"YOU WERE DOING 35.001 IN A
35 MILE ZONE..."



Improving the Teaching and Learning of Numeracy

Name: _____

School: _____



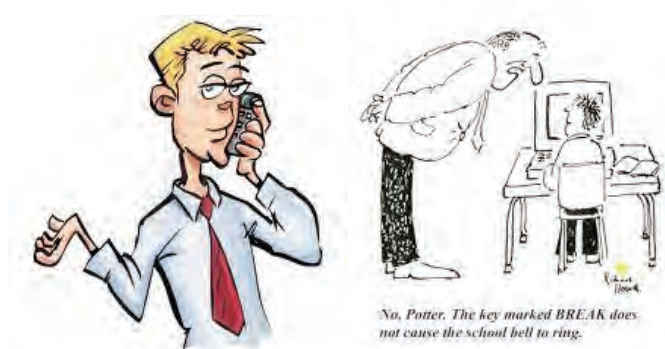
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What is your definition of numeracy?

Definition

Definitions for the term 'Numeracy'

On the one hand is an understanding of the scientific approach to the study of phenomena - observation, hypothesis, experiment, verification. On the other hand, there is the need in the modern world to think quantitatively, to realise how far our problems are problems of degree even when they appear as problems of kind" (Crowther, 1959, 270).

"We would wish the word 'numerate' to imply the possession of two attributes. The first of these is an 'at-homeness' with numbers and an ability to make use of mathematical skills which enables an individual to cope with the practical mathematical demands of his everyday life. The second is an ability to have some appreciation and understanding of information which is presented in mathematical terms, for instance in graphs, charts or tables or by reference to percentage increase or decrease" (Cockcroft, 1982, 11).

"...numeracy nonetheless nourishes the entire school curriculum, including not only the natural, social and applied science but also language, history and the fine arts" (Steen, 1999,8).

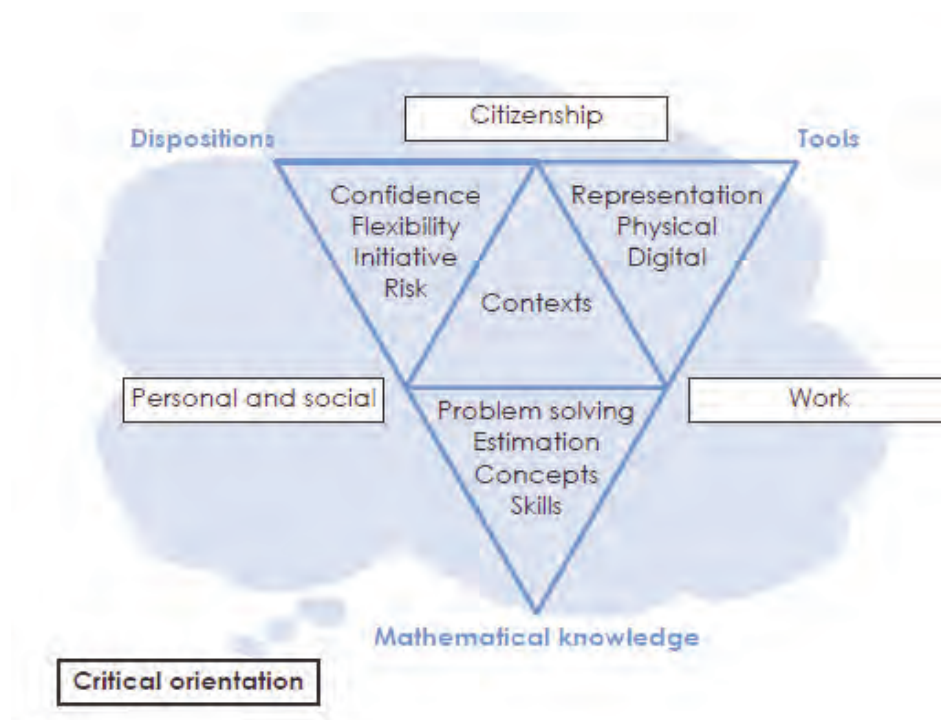
"Numeracy is not limited to the ability to use numbers, to add, subtract, multiply and divide. Numeracy encompasses the ability to use mathematical understanding and skills to solve problems and meet the demands of day-to-day living in complex social settings. To have this ability, a young person needs to be able to think and communicate quantitatively, to make sense of data, to have a spatial awareness, to understand patterns and sequences, and to recognise situations where mathematical reasoning can be applied to solve problems" (DES,2011,4)

"An individuals' capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals in recognising the role that mathematics plays in the world and to make the well-founded judgements and decisions needed by constructive, engaged and reflective citizens" (OECD, 2014, 28).

"At one extreme, numeracy is interpreted as equipping school leavers with the basic skills of arithmetic, and so numeracy is regarded as a subset of mathematics. At the other extreme, it is argued that the teaching of numeracy should address the political dimensions of mathematics and its uses in society, so that learners are supported in becoming critical citizens" (Askew, 2015,707).

"Despite different terminologies—mathematical literacy, quantitative literacy, quantitative reasoning, numeracy— there are similarities in the ideas canvassed. All require mathematical skills and reasoning to be applied to some problem set in a social environment. The disposition to use mathematics and capacity to mathematise are implied requirements" (Callingham et al., 2015, 550).

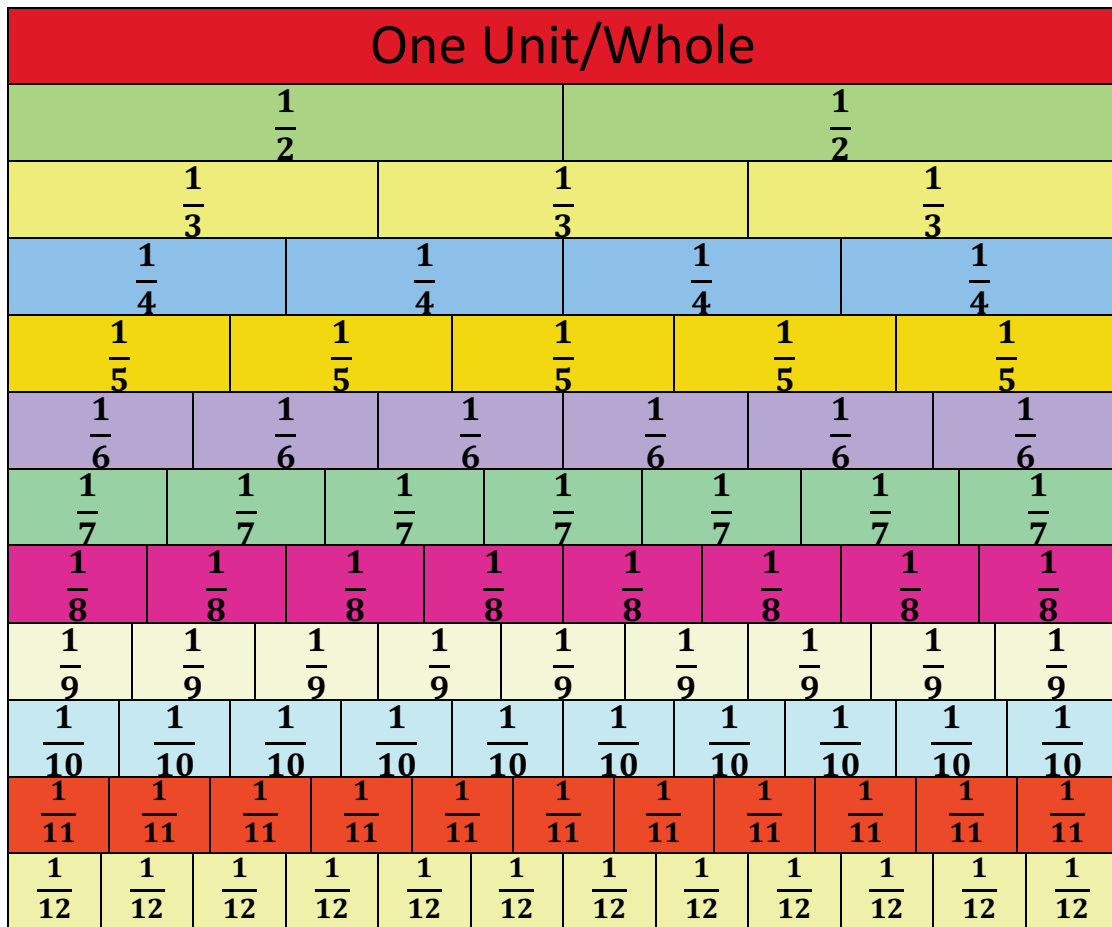
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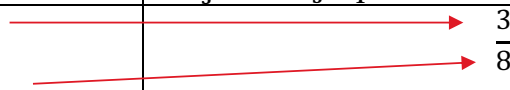
(Goos et al., 2012)

Use the model above to give an example of how numeracy is taught in any topic of any subject you teach.

Fractions



Key Vocabulary

Fraction	Compares a part to the whole. e.g. $\frac{1}{3}$ of €9 = $(\frac{1}{3} \times \frac{9}{1}) = €3$
Numerator = 3	 $\frac{3}{8}$
Denominator = 8	
Equivalent Fractions	Equivalent fractions are fractions whose numerator and denominator are in the same ratio as that of the original fraction. e.g. $\frac{1}{3}$ is equivalent to $\frac{2}{6}$

Common Approach to Fractions

Diagram

Estimate

Calculate

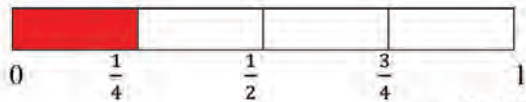
Problem 1. If Donald ate $\frac{2}{3}$ of his own bar of chocolate and if he ate $\frac{1}{4}$ of Tim's chocolate bar. What fraction of a bar of chocolate did he eat?

Diagram

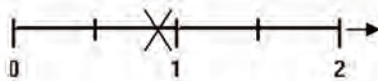


What fraction remains?

The CARS Checklist (Wray, 2007)



Estimate



Since the sum of $\frac{2}{3}$ and $\frac{1}{4}$ on the fraction strip above is less than one, represent the sum of the two fractions with an X on the number line.

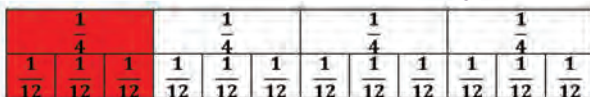
Calculate

Because $\frac{2}{3}$ and $\frac{1}{4}$ have different denominators, look for the common denominator of both fractions with the help of the fraction wall, i.e. what is the **lowest common denominator** that 3 and 4 divide into? When the common denominator is found, **add like terms to like terms**.



Now, what fraction remains?

$\frac{2}{3}$ is equivalent to $\frac{8}{12}$



$\frac{1}{4}$ is equivalent to $\frac{3}{12}$



$\frac{8}{12} + \frac{3}{12} = \frac{11}{12}$ of one bar of chocolate

Percentages

One Whole-100%									
50%					50%				
$33\frac{1}{3}\%$			$33\frac{1}{3}\%$			$33\frac{1}{3}\%$			
25%		25%		25%		25%		25%	
20%		20%		20%		20%		20%	
$12\frac{1}{2}\%$	$12\frac{1}{2}\%$	$12\frac{1}{2}\%$	$12\frac{1}{2}\%$	$12\frac{1}{2}\%$	$12\frac{1}{2}\%$	$12\frac{1}{2}\%$	$12\frac{1}{2}\%$	$12\frac{1}{2}\%$	$12\frac{1}{2}\%$
10%	10%	10%	10%	10%	10%	10%	10%	10%	10%

Key Vocabulary

Percentage	→	A number as a fraction of 100
13%	→	is equivalent to $\frac{13}{100}$

Whole School Approach

Express students' test scores as a fraction and then as a percentage

1. Make a fraction by placing the test score received over the total number of marks.
2. Estimate the percentage mark from the fraction.
3. Change the fraction to a percentage by multiplying the fraction by $\frac{100}{1}$.
4. Check your answer using a calculator.

Example 1. Marie got a score of 47 in her biology test out of a total possible score of 60. What percentage mark did Marie achieve in her exam?

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1. Fraction

Put the mark Marie achieved over the total number of marks for the exam. E.g. Fraction = $\frac{47}{60}$

At this point, ask could another fraction equivalent to $\frac{47}{60}$ be formed? In some cases there may be a common factor between the numerator and the denominator. In such cases express the fraction in its simplest form.

Estimate

Calculate

Check

2. Estimate

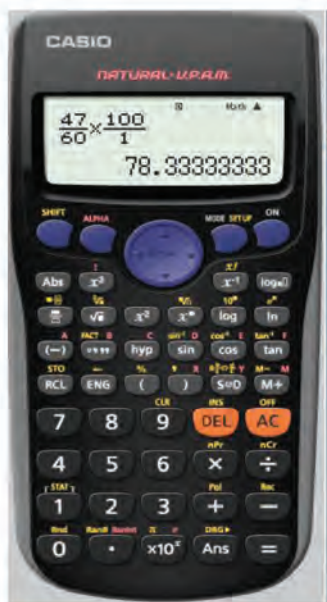
$\frac{30}{60}$ is equivalent to a $\frac{1}{2}$, which is equivalent to 50%. Therefore the answer will be greater than 50% because $\frac{47}{60}$ is greater than a $\frac{1}{2}$.

3. Calculate

$$\frac{47}{60} \times \frac{100}{1} = \frac{47}{(20)(3)} \times \frac{(20)(5)}{1} = \frac{47}{3} \times \frac{5}{1} = \frac{235}{3} = 78\frac{1}{3}\%$$

Since 20 is the highest common factor of both 60 and 100, we can simplify these two numbers before multiplying.

4. Check



To check the answer on the calculator, key in the following:



Is $78\frac{1}{3} = 78.3\bar{3}$? Explain your answer.

Explain how you could round Marie's percentage mark to the nearest whole number.

To calculate the percentage of a number

'of' means multiply

Problem 2: Sean earns €15 per week for walking his neighbour's dog. His neighbour increased his wage by 20%. How much per week did Sean's wage increase by?

Estimate

Calculate

Check

1. Estimate

20% is less than a $\frac{1}{2}$. Therefore, the answer will be less than $\frac{1}{2}$ of €15. i.e. $(\frac{1}{2} \times \frac{15}{1}) = €7.50$

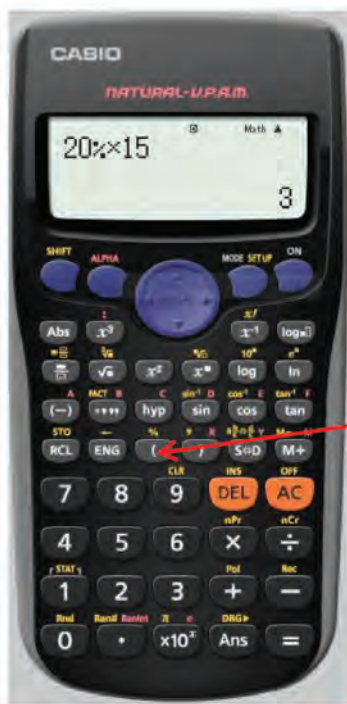
2. Calculate

$20\% = \frac{20}{100}$ which is equivalent to $\frac{1}{5}$.

$\frac{1}{5}$ of €15 means: $\frac{1}{5} \times \frac{15}{1} = \frac{1}{(5)(1)} \times \frac{(5)(3)}{1} = \frac{1}{1} \times \frac{3}{1} = \frac{3}{1} = €3$

Since 5 is the highest common factor of both 5 and 15, we can simplify these two numbers before multiplying

3. Check



To check the answer on the calculator, key in the following:

20

SHIFT

(

x

15

=

How did the estimate compare to your final answer? Explain your findings.

Decimals

Why do we need decimals?

- Decimals give a statement of the accuracy of measurement
- They are convenient to show relative size

What is the difference between 3.2 and 3.20?

- 3.2 indicates a level of accuracy of $\frac{1}{10}$, i.e. 3.2 is between 3.15 and 3.24
- 3.20 indicates a level of accuracy of $\frac{1}{100}$, i.e. 3.20 it is between 3.150 and 3.249

Why is $312 > 289$?

To answer the question we need to understand the place value system.

hundreds	Tens	Units
3	1	2

hundreds	Tens	Units
2	8	9

Because the $3 > 2$ in the hundreds, $312 > 289$

A common misconception with decimals is that students see the number with the most digits as the largest number. E.g $2.127 \neq 3.2$

It is essential that students fully understand the place value system.

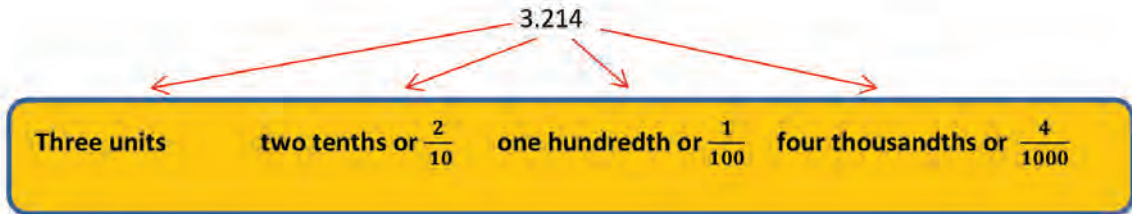
The size of a decimal is determined by the size of its digits, on a sliding scale, from left to right.



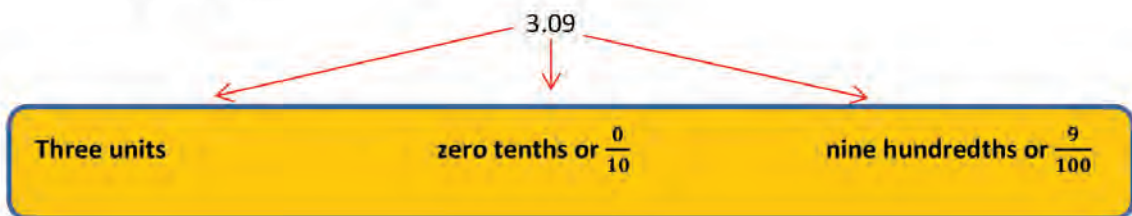
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Hundreds	tens	units	.	tenths	hundredths
----------	------	-------	---	--------	------------

Write the following decimals in order of size: 3.214, 3.09, and 3.8.



$$3.214 = 3 + \frac{2}{10} + \frac{1}{100} + \frac{4}{1000}$$



$$3.09 = 3 + \frac{0}{10} + \frac{9}{100}$$



$$3.8 = 3 + \frac{8}{10}$$



Written in order of size, from smallest to largest are: 3.09, 3.214, 3.8

Adding and Subtracting Decimals

Simplify $0.5 + 1.34$

$$\begin{array}{r} 0.50 \\ + 1.34 \\ \hline 1.84 \end{array}$$

In this example we've obeyed the rule of adding like terms to like terms.
How do we know which terms are the like terms?

Simplify the following: $0.8 - 0.008$

$$\begin{array}{r} 0.800 \\ - 0.008 \\ \hline 0.792 \end{array}$$

What is the value of the 8 in 0.8?

What is the value of the 8 in 0.008?

Note

Addition and subtraction of decimals is similar to that of fractions in that we add the numbers with the same denominator. (i.e. like terms to like terms)

e.g. $\frac{1}{10} + \frac{1}{10} = \frac{2}{10}$, which is equivalent to $\frac{1}{5}$.

Also, $0.1 + 0.1 =$

$$\begin{array}{r} 0.1 \\ + 0.1 \\ \hline 0.2 \end{array}$$

Algebraic fractions are also added and subtracted in the same manner as fractions and decimals. (i.e. like terms to like terms)

Multiplying Decimals

Estimate

Calculate

Check

Example: 34.5×20.5

Estimate

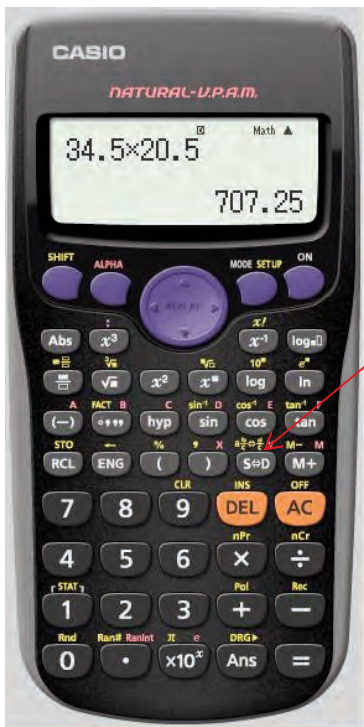
2 times 35 is 70, so 20 times 35 is 700. A reasonable estimate of the answer is 700.

Calculate

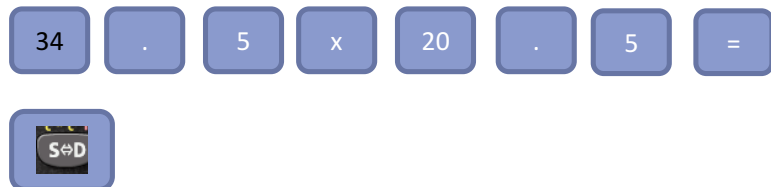
34.5
×20.5
1725
69000
707.25

The common problem with multiplication in decimals is where to place the decimal point. If the original estimate is accurate this should inform the decision. In this case the point goes between the 7 and the 2, giving each digit the correct value.

Check



On the calculator insert:



Division of Decimals

Estimate

Calculate

Check

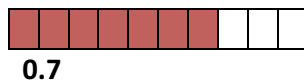
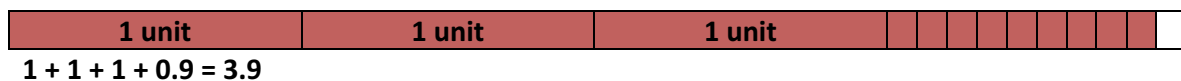
Divide 3.9 by 0.7

Estimate

By rounding to the nearest whole number, $4 \div 1 = 4$. A reasonable estimate to the answer is 4.

Estimation will not always give a very accurate result

Calculate



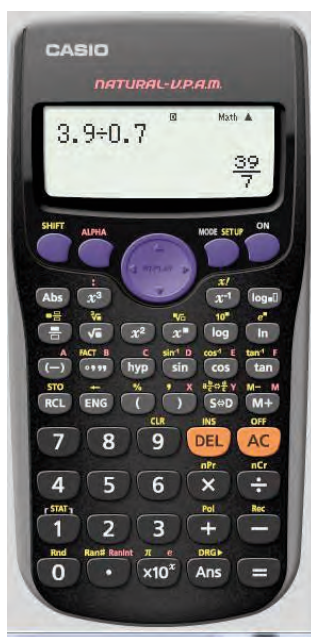
$$3.9 \div 0.7$$

$$39 \div 7 = \frac{39}{7} = 5 \frac{4}{7}$$

To divide by a decimal we bring the divisor to a whole number. In this example we multiply 0.7 by 10. To keep the calculation equal to what is asked we must also multiply 3.9 by 10.

Check

3	.	9	÷	0	.	7	=
---	---	---	---	---	---	---	---



Avoid common misconceptions:

Division does not always make a number smaller

Also

Multiplication does not always make a number bigger

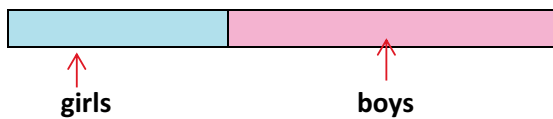
e.g. $6 \times 0.5 = ?$

Co-ordinated Approach to Ratio

A fraction compares a part to the whole

There are 15 girls and 18 boys in a particular class. What fraction of the total class are girls?

Diagram



Estimate

From the diagram it is clear that girls account for less than a $\frac{1}{2}$ of the total. Therefore the answer will be less than a $\frac{1}{2}$.

Calculate

The total (or the whole) in this case is the total number of students.

$$15 + 18 = 33.$$

The fraction of the class comprising of girls is

$$\frac{15}{33} = \frac{\cancel{3}(5)}{\cancel{3}(11)} = \frac{5}{11}$$

Check

Answer is in line with the estimate

To write a fraction in its simplest form, look to see if there is a highest common factor to the numerator and the denominator.

A ratio compares a part to a part

There are 15 girls and 18 boys in a particular class. What is the ratio of girls to boys in that class.

Estimate

From the figure above there are only a few more boys than girls. Therefore the ratio will be a far less than 1:2 (girls:boys) and closer to 1:1

Calculate

$$15:18 \\ \cancel{3}(5) : \cancel{3}(6) \\ 5:6$$

Ratio is written in its simplest form.

Look to see if the numbers in the ratio have a highest common factor.

Check

5:6 is in line with the estimate.

Avoid common misconceptions such as

$$5:6 \neq \frac{5}{6}$$

Graphical Analysis - A Common Approach for Bar Charts and Trend Graphs

When analysing bar charts and trend graphs it is good practice to describe what the graph is measuring. If there is no title you should give a description of what the two axes (i.e. the horizontal line and the vertical line) are representing.

- Two aspects of data analysis that are important to consider are the measures of spread of the data and the central tendency of the data.

Here is a toolkit for analysis:

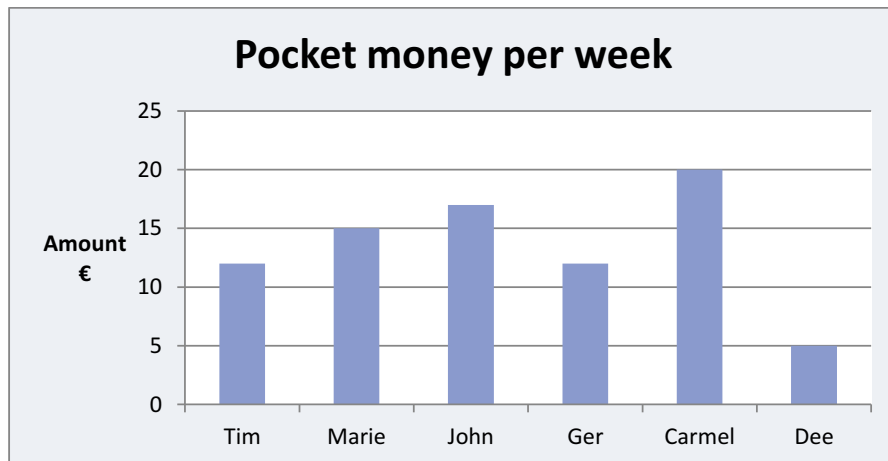
Measure of Spread

- **Range:** The difference between the largest value and the smallest value.
- **Standard Deviation:** The standard deviation measures the dispersion or spread of data from the mean. The smaller the standard deviation, the less widely dispersed (or spread) the data is.

Measure of Central Tendency

- **Mean:** The average number of the data. The mean is calculated by calculating the sum of all the numbers in the data and then dividing the sum by the number of numbers. In some cases the mean can be a poor measure of central tendency. For instance, if an 80 year old is included in the average age of a first year college class then the mean age will be higher as a result, not reflecting accurately the central tendency of the data.
- **Median:** The middle number when the data is arranged in order. The median is found by writing the array of numbers out in order of their size and then picking the middle number. If there is an even number of data points, the median is the average of the two middle points when ordered.
- **Mode:** The number that occurs most often. For example if there are five candidates in an election and first past the post is deemed elected, then the mode or the modal person is the elected candidate because they receive the most votes.

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Median of the data:

$$5, 12, 12, 15, 17, 20. \frac{12+15}{2} = \frac{27}{2} = €13.50$$

Mean of the data:

$$\frac{12+15+17+12+20+5}{6} = €13.50$$

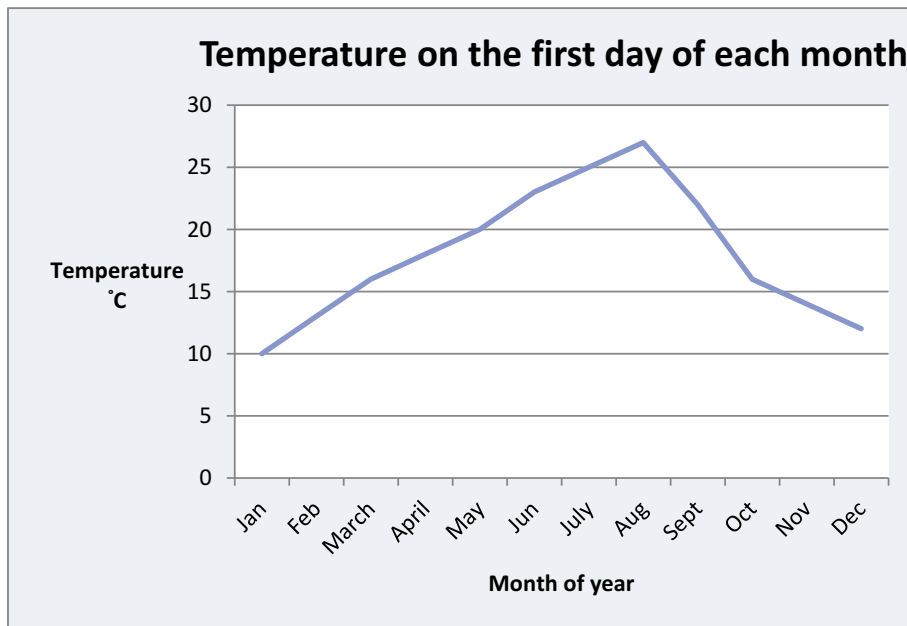
Mode of the data: €12

Sample Analysis

This bar chart represents the amount of pocket money per week received by six children. The range in pocket money is €15, with Dee receiving the least at €5, and Carmel receiving the most at €20. The mean amount of pocket money earned per week is €13.50.

When the standard deviation of a set of data is high, what limitations does this place on the mean, as a number that solely represents the central tendency of the data?

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Median of the data:

10,12,13,14,16,**16,18**,20,22,23,25,27

$$\frac{16+18}{2} = \frac{34}{2} = 17^{\circ}\text{C}$$

Mean of the data:

$$\frac{10 + 13 + 16 + 18 + 20 + 23 + 25 + 27 + 22 + 16}{12}$$

$$\text{Mean} = \frac{216}{12} = 18^{\circ}\text{C}$$

Mode of the data = 16°C

Sample Analysis

This trend graph represents the temperature on the first day of each month. The temperature range is 15°C, with the highest temperature recorded in August at 27°C and the lowest temperature recorded in January at 10°C. The mean temperature per month is 18°C.

A Co-ordinated Approach to Statistics across the Curriculum

Statistic	A numerical property associated with a sample of a population e.g. percentage of people in a poll who say they will vote “Yes” in a referendum
Parameter	A parameter is a numerical property associated with a population e.g. A response to a question in a census.
Variable	We measure its value for each person, plant, animal or thing and it varies (variable) from person to person e.g. the height of an individual or their favourite sport
Population	The entire collection (people, plants, animals, things) about which information is required e.g. The population of voters in a country is all those eligible to vote in an election in that particular country.
Sample	Any subset or smaller group of a population e.g. a representative subset of students from the school A sample should be random and representative of the population.
Primary Data	Data collected by user e.g. the compiler of the investigation
Secondary Data	Data collected by somebody other than the user e.g. results of investigations published in newspapers `
Bias	Inclination or prejudice for or against one person or group, especially in a way considered to be unfair.

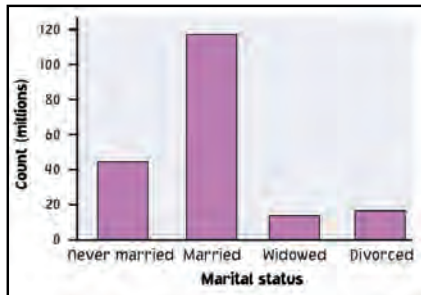
BIAS

7 questions to consider when you encounter statistical information in your subject area.

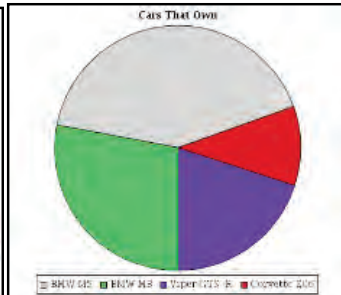
1. What was the purpose of the investigation?
 - ✓ How might this have influenced the investigation?
2. Was the sample representative?
 - ✓ Had everybody in the population an equal likely chance of being represented in the result?
3. Is the data recent?
4. What were the exact questions asked?
5. Could the results have been interpreted differently?
6. Is this the original investigation or a report based on the investigation?
7. Does the report accurately reflect the results of the investigation?
 - ✓ Have only parts of the investigation been reported?

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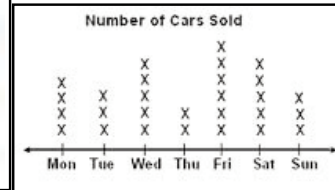
Visual - Graphical Representations



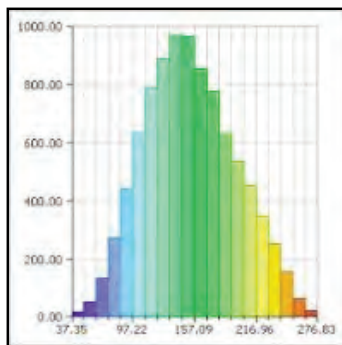
Bar Chart



Pie Chart

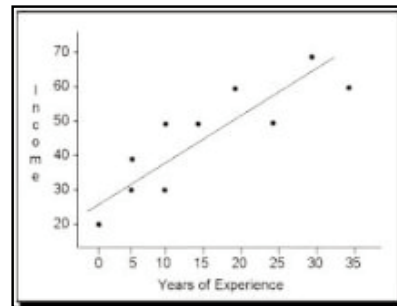


Line Plot

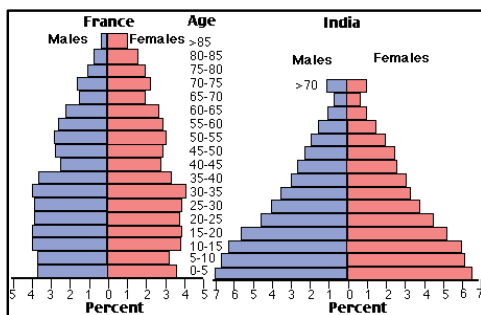


Histogram

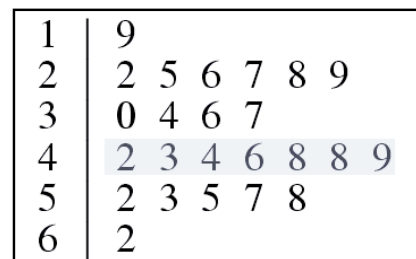
(Continuous Data e.g. height, hair length)



Scatter Plot (Relationship)



Population Pyramid



Stem and Leaf Plot Key: 1|9 = 19 cm

Mathematical Language

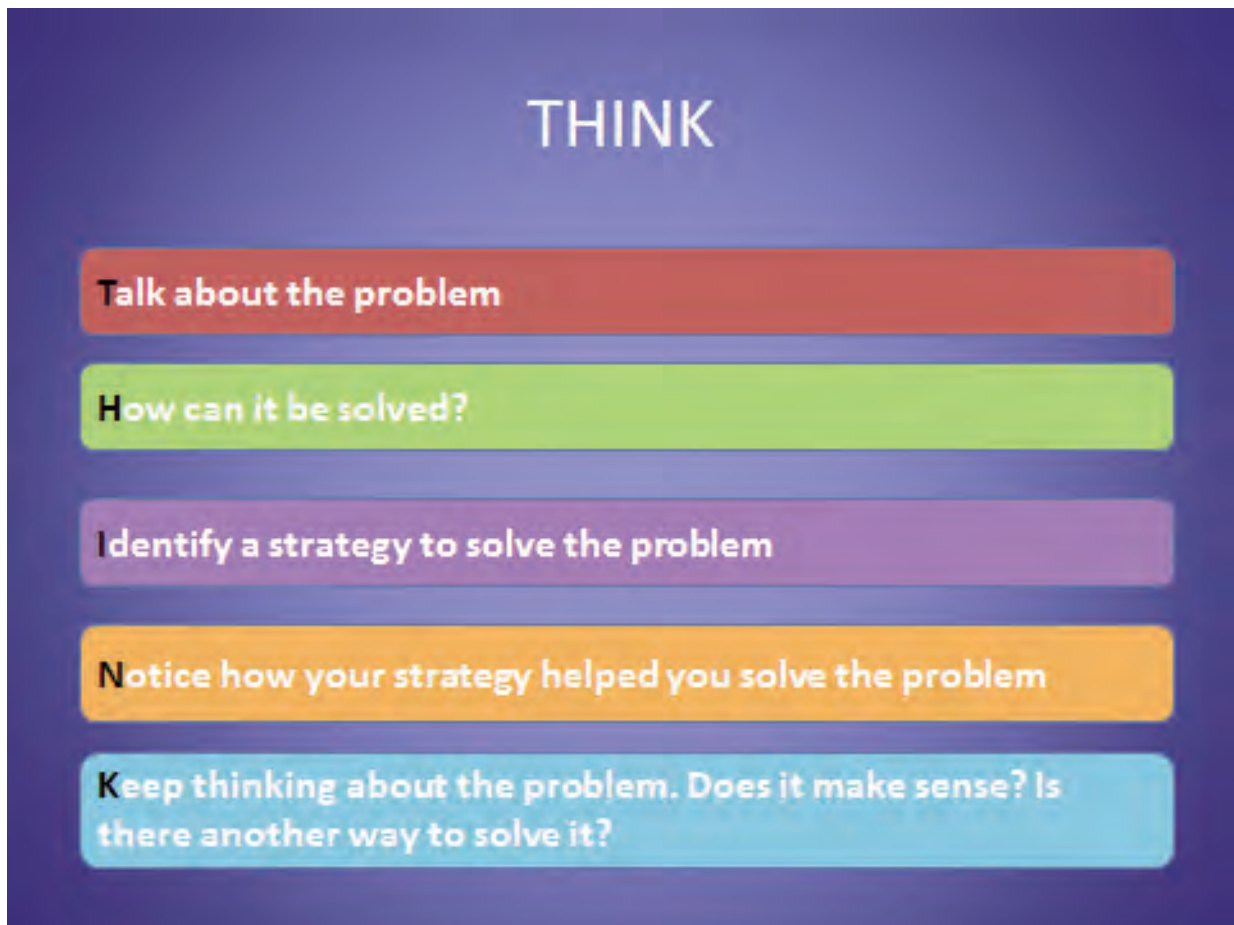
Word	EE	ME	Symbol

With thanks to Dr. Máire Ní Ríordáin

Mathematical Language

Definition:	Related Words:
Write a statement in mathematical English including the word:	The word looks like the graphically/symbolically:

Problem-Solving Strategy



(Thomas, 2006)

How did you evaluate your answer?

How did you decide what to do?

What did you do that helped you understand the problem?

Can something you did here help you solve other problems?

Classroom Discussion Approach to Problem Solving

Did you find any information you didn't need? How did you know that the information was not important?

Did you try something that didn't work? How did you figure out it was not going to work out?

Was there something in this problem that reminded you of another problem?

(Van de Walle et al., 2013)

Tic-Tac-Toe (Extension Tasks)

Verify that you are correct	Solve this question using a different method	Design a different problem that would give the same answer
Identify a pattern, generalise or create a proof from your work	Identify any situations where this method would not work?	Find an example of where you might apply this problem in another subject/ work/life situation
What did you learn? How did you learn?	Design a revision guide for this topic	Create a mind map of the activity

Useful Websites

- <http://nlvm.usu.edu> Resources for the various strands available-Scroll down to grades (9-12)
- <http://www.thatquiz.org/> Very good quizzes against the clock
- <http://www.mathplayground.com/geoboard.html> Interactive geoboard
- <http://www.topmarks.co.uk/PlayPop.aspx?f=NumberLinev5> Interactive number line
- <http://illuminations.nctm.org/ActivityDetail.aspx?ID=40> Tower of Hanoi puzzle
- www.scoilnet.ie 388 resources for Post-Primary Maths
- <http://nrich.maths.org/9196> Great source for problems
- <http://www.theteacherscorner.net/> A collection of educational worksheets, lesson plans, activities and resources for teachers and parents
- <https://www.mangahigh.com/en-gb/> A free games-based maths teaching approach
- www.amathsdictionaryforkids.com –good for Maths definitions, posters and problemsolving.

Useful Apps

- Kings of Maths: Very good for mental arithmetic
- Maths Quiz Gameshow: Quizzes on various maths topics
- Digiwhiz: Very good for number fluency
- 5 Dice: Exercises combining five numbers and operations to reach a target number
- Wolfram Algebra-Course Assistant: Multi-representational approach to algebra
- Maths Bingo: Good for basic arithmetic
- Algebra Touch: Demonstration of basic rules of addition/subtraction
- Fraction Basic: Lessons based on the different aspects of fractions
- Tangrams: app for puzzles and shape
- NumbersGame: An app which combines foreign languages and the numbers 1-10
- Math Gr5: topics such as number and operations, fractions and measuring etc are included
- Wolfram Pre-Algebra Course Assistant: Multi-representational approach to algebra
- Conundra Math: Exercises combining three numbers and operations to reach a target number



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